Stellar Surface Imaging of LO Pegasi via Light-curve Inversion
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Introduction
Sunsprts are regions of strong magnetic field (~2-3 kG) that suppress convection on the surface of the Sun, causing those areas to be cooler and thus darker than their surroundings. We studied analogous phenomena called sunspots on the star LO Pegasi. Because stars are imaged as mere pinpoints, we used an indirect mathematical technique called light-curve inversion (LI) to map the sunspots (Harmon & Crews 2000, AJ, 120, 3274). We acquired BVRI images of LO Pegasi at Perkins Observatory and performed differential aperture photometry to generate light curves. The spots cause changes in the star’s brightness as it rotates, which we analyzed using LI.

Calculating the Light Curve from the Surface

- We divide the stellar surface into spherical rectangular patches of approximately equal areas.
- We index the patches such that patch (i,j) is the pth patch in the ith latitude band, and the stellar surface has N latitude bands with Mp patches in the ith latitude band.
- We let M_Ω sinθ, where θ is the co-latitude of the ith latitude band, so that all the patches will have approximately equal areas subject to the constraint that M_ must be an integer.
- At the time t_{ab} of observation number k through filter n, the intensity of the star at Earth is given by:

I_{ab} = \sum_{i=1}^{N} \sum_{j=1}^{M} \Omega_{ijk} I_{ab}^{\nu} \Omega_{ijk}^{\nu},

where \Omega_{ijk}^{\nu} is the specific intensity along the outward normal of patch (i,j) as observed through filter n. One filter is chosen as the primary and assigned n = 1, and we set J_{ij} = J_{ij}^{\nu}, I_{ab}^{\nu} is obtained from Ω_{ijk}^{\nu} from the assumed spot and photosphere temperatures.

\Omega_{ijk}^{\nu} is the solid angle subtended by patch (i,j) at time t_{ab} as seen from Earth. We are only concerned with relative brightness, so we set \Omega_{ijk}^{\nu} equal to the projected area perpendicular to the line of sight of patch (i,j) and treat the star as a unit sphere. (We set \Omega_{ijk}^{\nu} = 0 if the patch is on the far side of the star.)

I_{ab}^{\nu} is the limb darkening of patch (i,j) at time t_{ab}, \lambda_{avg}^{\nu} is the ratio of the specific intensity emitted along the line of sight to that emitted along the outward normal as seen through filter n.

Calculating the Surface from the Light Curve

While it would be a simple matter to calculate the light curve if we knew the surface brightness distribution, we must solve the inverse problem of finding the surface brightness distribution from the light curve data. One would think we should just find the set of J_{ij} that gives the best fit to the data. However, if a star were peppered with many small bright and dark spots, the result would be a high frequency ripple in the star’s light curve. Our noise in data also takes the form of a high frequency ripple, so if we try to fit to the data too well we will generate a star peppered with bright and dark spots that are non-physical noise artifacts. This extreme sensitivity to noise makes the problem ill-posed. To overcome this, we define the objective function E via

E(J;λ, B) = λS(J; B) + G(J),

where J represents the set of all J_{ij}, S is the smoothing function, λ is the smoothing parameter, and G is the goodness-of-fit function. We define G via

G(J) = \frac{(2.5 \log_{10} e)^2}{P} \sum_{n,I} \sum_{g} \frac{1}{\Omega_{ijk}^{\nu} \Omega_{ij}^{\nu}} \left( \frac{I_{ab}^{\nu} - I_{ab}^{\nu}}{J_{avg}^{\nu}} \right)^2.

Here \Omega_{nj}^{\nu} is the measured intensity for the kth data point of the light curve through filter n; gc is the estimated variance expressed in magnitudes between the measured and calculated light curve intensities; P_{n} is the number of points in light curve n; P is the combined number of points in all the light curves; and Q is the number of light curves. For S we use

S(J; B) = \sum_{n,I} \sum_{g} c_{g} (\tilde{J}_{g} - J_{avg})^{2} / \sum_{n,I} M_{n},

c_{g} = \begin{cases} 1 & \text{if } J_{g} \leq J_{avg} \\ B & \text{if } J_{g} > J_{avg} \end{cases}

Our goal is to find the set of J_{ij} that minimizes E. When G is small the difference between the measured light curve and the reconstructed light curve is minimized, i.e. the fit to the data is optimized.

The smoothing function S rejects surfaces that are peppered with many small spots because for such a surface the difference J_{ij} - \tilde{J}_{ij} will be large for many of the patches.

The bias parameter B (which is greater than 1) favors surfaces that have a small number of dark patches (that represent the spots) and many patches slightly brighter than average (that represent the photosphere). This is because as patches are “folded” to deviate from the average in order to fit the light curve data, the “penalty” (increase in E) for a patch being brighter than average by a given amount is B times greater than if it were darker than average by the same amount, so minimization of E favors using dark patches to fit the data.

λ is a Lagrange multiplier that strikes a balance between goodness-of-fit (G) and smoothness of solution (S). As λ→∞, the surface that provides the minimum E is completely featureless with poor fit to the data. As λ→0, the surface that provides the minimum E is a surface with the best fit to the data but will be peppered with noise artifacts.

The task of MLI is to find the J_{ij} along with the optimal λ (\lambda_{opt}), and optimal B (B_{opt}). See below for the procedure for the MLI.

Data Collection and Inversion

- Data were collected by measuring the intensity of the star through B, V, R and I filters via aperture photometry. We then plotted intensity versus the star’s rotational phase to produce a light curve for each filter.
- LI then simultaneously inverted all four light curves designating one light curve as the primary and then scaling results for the others.
- The reason for using multiple filters is that the limb darkening is different through different filters, and this allows LI to better determine the latitudes of spots.

Results


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Procedure Used by MLI

Start with an initial B and λ.

Minimize E(J;λ, B) by varying the J_{ij}.

Calculate f for the solution, J_{opt}.

f(λ, B) = \sqrt{\max \left( E(J_{opt}; λ, B) \right)} - 1

If f > 0 (within tolerance), make new guess for \lambda_{opt}.

\lambda_{opt} = \arg \min_{\lambda} \left( E(J_{opt}; λ, B) \right)

If f = 0 (within tolerance), stop.

\sigma^{2} is the estimated variance of the darkest spot to the photosphere intensity expressed in magnitudes.

\frac{J_{spot}}{J_{photphere}} is the ratio of estimated intensity of the darkest spot to the photosphere intensity based on other studies and observations of the star.

It can be shown that G = 1 (so that f = 0) when the overall residual between the calculated and data light curves is equal to the estimated noise variance. We also want the ratio of the brightness of the darkest patch to the average patch brightness on our reconstructed surface to be the same as our estimate of the ratio of spot brightness to photosphere brightness on the actual star, so we want to have g = 0 as well.

*Result:* The final step gives a smooth stellar surface where the goodness-of-fit of the reconstructed light curve to the data light curve is neither better nor worse than is justified by the noise in the data (thus avoiding noise artifacts in the solution), and the reconstructed spot intensity to photosphere intensity ratio is approximately what we estimate it should actually be.