Josephson's Explanation of Physics of Superconducting Tunnel Junctions—What is a Josephson Junction (JJ)?

Using conservation of charge, when the current flows entirely through the JJ, the current equals the sum of the three channels.

A Simple Physical Model for JJs, Resistively-Capacitively-Shunted-Junction (RCSJ) Model

The RCSJ model consists of three different channels:

- **S**: Super Current Channel
  \[ I_S = I_C \sin \phi \]
- **R**: Resitively Channel
  \[ I_R = \frac{V}{R} \]
- **C**: Capacitance Channel
  \[ C \frac{d\phi}{dt} = \frac{Q}{C} \]

When \( I_B < I_C \), the current flows entirely through the Super current channel and \( \phi = 0 \).

When \( I_B > I_C \), the bias current equals the sum of the three channels.

Using conservation of charge, \( J_B \) can be represented through a single governing equation

\[ J_B = I_C \sin \phi + \frac{h}{2e} \frac{d\phi}{dt} + \frac{hC}{2e} \frac{d^2\phi}{dt^2} \]

Why are we Interested in JJs and Synchronization of JJs?

When a JJ is biased with a current greater than its critical current, a voltage drop across the JJ appears and the JJ emits microwaves with power measurable in nanowatts.

We have been studying the synchronization of several JJs called an array. Such an array could potentially emit microwave power in the microwave range or higher and could possibly emit in a region of the electromagnetic spectrum, for which no natural sources are known to exist. One possible example, the high \( T_C \) superconductor \( Bi_2Sr_2CaCu_2O_8+1 \) (see image below).

Determining Level of Synchronization, Exhibited by a Simple Array of JJs: A Single Plaquette

From conservation of charge applied to the 4 superconductors comes a system of dimensionless equations for the plaquette. (neglect capacitance)

\[ i_B = I_C \sin \phi \]
\[ i_B = \frac{h}{2e} \frac{d\phi}{dt} \]
\[ i_B = \frac{hC}{2e} \frac{d^2\phi}{dt^2} \]

The sum of the phases around a closed loop equals zero.

\[ \phi_1 + \phi_2 + \phi_3 + \phi_4 = 0 \]

Using Mathematica, this system of coupled differential equations is numerically solved for phases and voltages as a function of time. These results can be used to calculate the phase order parameter. The phase order parameter quantifies the degree of synchronous oscillations in an array.

Process for calculating phase order parameter:

\[ \tau = \frac{1}{N} \sum_{i=1}^{N} \frac{d\phi_i}{dt} \]

(definition of phase order parameter)

\[ \langle \phi \rangle = \frac{1}{\Delta \tau} \int \phi \sqrt{\Delta \tau} d\tau \]

(averaged value)

\( \tau \) is chosen so that transient behavior is completed
\( \Delta \tau \) is chosen to sample many cycles of voltage oscillations

\[ 0 \leq |\langle \phi \rangle| \leq 1 \]

| \( |\langle \phi \rangle| = 1 \) | perfect phase synchronization

The phase order parameter is calculated for three geometries as a function of such parameters as bias current, junction critical current, and coupling strength, \( \Omega \).

For each of the three geometries, our goal is to find the optimal combination of junction parameters such that the largest possible subset of JJs in the array oscillates synchronously.

Conclusion

We identify phase synchronization for three geometries of Josephson Junction arrays.

- For a single plaquette, we identify a range of bias current and coupling strength values that produce phase synchronization of the two-voltage active junctions.
- For a double plaquette with a shunt junction, we identify a range of shunt junction critical current, bias current, and coupling strength values that produce phase synchronization of all four voltage-active junctions.
- For a serial array with a shunt junction, we identify a range of bias current and shunt junction critical current values that produce phase synchronization of the two serial junctions.

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