Stellar Surface Imaging of LO Pegasi via Light-Curve Inversion

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Project Goal
Our goal was to map the surface of LO Pegasi in order to characterize the dark starspots on its surface. The method we used is Light-Curve Inversion, which exploits the variations in the star’s brightness produced as the spots are carried into and out of view by the star’s rotation.

LO Pegasi (BD +22°4409) is a K5-7 main sequence star with rotational period 10.17±0.08 h (Lister, et. al 1999, MNRAS, 307, 685).

Procedure
Images were obtained at Perkins Observatory using a 0.2-m Meade LX200 telescope equipped with an SBIG ST-8XE CCD camera and Optec B, V, R and I filters. Differential aperture photometry was then performed to generate light curves. Using multiple filters improves the latitude resolution of the maps by taking advantage of differing degrees of limb darkening as seen through the different filters.

The Light-Curve Inversion Algorithm
The algorithm reconstructs the target star’s surface by modeling it as a sphere subdivided into \( N \) bands in latitude, with the \( 1 \)st band surrounding the visible pole. By convention, zero longitude is the meridian which intersects the equator at the limb on the approaching side of the star at the time of the first observation. The \( P \)th latitude band is subdivided into \( M_p \) patches in longitude numbered from the zero meridian such that the areas of all the patches on the surface are nearly equal. Denoting the specific intensity along the outward normal of patch \((i,j)\) by \( I_{ij} \), the intensity \( I_k \) at Earth at observation time \( t_k \) is given approximately by

\[
I_k = \sum_{i=1}^{N} \sum_{j=1}^{M_i} \Omega_{ij} L_{ij} J_{ij},
\]

where \( \Omega_{ij} \) is the solid angle subtended by the patch at time \( t_k \), and \( L_{ij} \) is the limb darkening evaluated at the center of the patch at this time. This expression becomes exact in the limit of an infinite number of patches.

We cannot simply find the set of \( J_{ij} \) which provide the best fit to the light curve data in the least-squares sense, because the problem is ill-posed; the algorithm would introduce spurious small spots in an attempt to reproduce the small brightness variations due to noise. Instead we perform a constrained non-linear inversion which imposes a smoothness constraint on the solution to suppress spurious noise artifacts. We find the set of \( J_{ij} \) which minimizes the objective function

\[
E(J,I,\lambda,B) = \lambda S(J,B) + G(J,I).
\]

Here \( G \) is a function which measures the goodness-of-fit between the data light curve and the light curve of the reconstructed surface; \( S \) is a function which expresses the “smoothness” of the reconstructed surface, with smaller values of \( S \) corresponding to “smoother” surfaces; \( \lambda \) is the smoothing parameter, and \( B \) is the bias parameter (see below). For \( \lambda \to 0 \) we minimize \( G \), producing too good a fit to the data in that noise is inverted, producing artifacts. For \( \lambda \to \infty \) we minimize \( S \), producing a featureless solution and a very poor data fit. The algorithm finds the value of \( \lambda \) such that \( G \) is equal to a corresponding estimate of the noise level in the data. We thereby find the “smoothest” solution which fits the data to no greater a degree than is justified by the noise (e.g., Twomey 1977, Introduction to the Mathematics of Inversion in Remote Sensing and Indirect Measurements).

The goodness-of-fit function used in this study is

\[
G(J,I) = \frac{1}{Q} \sum_{n=1}^{Q} \frac{1}{P_n \sigma_n^2} \left( (2.5 \log_{10} e)^3 \sum_{k=1}^{P_n} \left( \frac{I_{nk} - \hat{I}_{nk}}{I_{nk}} \right)^2 \right).
\]

Here \( I_{nk} \) is the observed intensity for the \( k \)th observation through the \( n \)th filter, \( \hat{I}_{nk} \) is the corresponding intensity for the reconstructed surface, \( P_n \) is the number of data points in the light curve for filter \( n \), \( \sigma_n \) is the estimated RMS noise in magnitudes for filter \( n \), and \( Q \) is the number of filters. The term in curly brackets represents the variance between the data and reconstructed light curves for the \( n \)th filter expressed in magnitudes.

The smoothness function used here is

\[
S(J,B) = \sum_{j=1}^{M_j} \sum_{i=1}^{N} c_i (J_{ij} - J_{avg})^2 / \sum_{j=1}^{M_j} \sum_{i=1}^{N} c_i,
\]

where \( J_{avg} \) is the average value of \( J_{ij} \), and where \( c_i = 1 \) if \( J_{ij} < J_{spot} \), while \( c_i = B \) if \( J_{ij} > J_{phot} \). For \( B > 1 \) this biases the solution towards dark spots on a nearly uniform photosphere. \( B \) is chosen so that \( \lambda_{spot}/\lambda_{avg} = J_{spot}/J_{phot} \), where \( J_{spot} \) and \( J_{phot} \) are the spot and photosphere intensities based on their temperatures.

Results
For this study two sets of observations were inverted. The first set was gathered from July 7-9, 2007, and the second set was gathered on July 22 and 23, 2007.

At top are the reconstructed surface and light curves for July 7-9, while the results for July 22-23 are at bottom. Both reconstructions show a prominent spot centered at \( \pm 30^\circ \) latitude. The apparent longitude drift of \( \pm 20^\circ \) is consistent with systematic error due to the uncertainty in the rotation period. The somewhat poorer fits to the data on July 22-23 may mean that our estimates of the spot and photosphere temperatures (3500 K and 4250 K) should be revised. We are investigating this possibility.

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